An Introduction to the Stability of Dynamical Systems

The present 12 hour seminar will introduce some basic concepts related to the stability analysis of dynamical systems disturbed away from their equilibrium states. Despite the generality of the topic, this seminar will be anchored in the field of fluid mechanics and heat transfer. First, a historic perspective is presented, highlighting the origins of stability analysis in celestial mechanics and its early transition into fluid mechanics. Second, equilibrium states are presented, and their required accuracy constraints are highlighted. Some of the most well-known tools used obtained these states are also presented. Third, the linear and nonlinear disturbance behavior is introduced, where dynamical systems are disturbed away from their equilibrium states by small and large perturbations. They are discussed in the context of subcritical and supercritical bifurcations from stability. Fourth, modal and non-modal linear analysis is presented. They are used to introduce the notion of the time asymptotic disturbance behavior, governed by their eigenvalues, as well as the transient disturbance behavior, governed by their eigenvectors. Finally, local and global disturbance behavior is discussed in the context of spatially developing flows. They are used to introduce the concepts of convective, absolute and global instabilities. All these topics are illustrated with simple model problems, used to highlight important mathematical steps, as well as real world problems from the literature, used to highlight practical applications.